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## PREFACE

*Fundamentals of Finite Element Analysis* is intended to be the text for a senior-level finite element course in engineering programs. The most appropriate major programs are civil engineering, engineering mechanics, and mechanical engineering. The finite element method is such a widely used analysis-and-design technique that it is essential that undergraduate engineering students have a basic knowledge of the theory and applications of the technique. Toward that objective, I developed and taught an undergraduate “special topics” course on the finite element method at Washington State University in the summer of 1992. The course was composed of approximately two-thirds theory and one-third use of commercial software in solving finite element problems. Since that time, the course has become a regularly offered technical elective in the mechanical engineering program and is generally in high demand. During the developmental process for the course, I was never satisfied with any text that was used, and we tried many. I found the available texts to be at one extreme or the other; namely, essentially no theory and all software application, or all theory and no software application. The former approach, in my opinion, represents training in using computer programs, while the latter represents graduate-level study. I have written this text to seek a middle ground.

Pedagogically, I believe that training undergraduate engineering students to use a particular software package without providing knowledge of the underlying theory is a disservice to the student and can be dangerous for their future employers. While I am acutely aware that most engineering programs have a specific finite element software package available for student use, I do not believe that the text the students use should be tied only to that software. Therefore, I have written this text to be software-independent. I emphasize the basic theory of the finite element method, in a context that can be understood by undergraduate engineering students, and leave the software-specific portions to the instructor.

As the text is intended for an undergraduate course, the prerequisites required are statics, dynamics, mechanics of materials, and calculus through ordinary differential equations. Of necessity, partial differential equations are introduced but in a manner that should be understood based on the stated prerequisites. Applications of the finite element method to heat transfer and fluid mechanics are included, but the necessary derivations are such that previous coursework in those topics is not required. Many students will have taken heat transfer and fluid mechanics courses, and the instructor can expand the topics based on the students’ background.

**Chapter 1** is a general introduction to the finite element method and includes a description of the basic concept of dividing a domain into finite-size subdomains. The finite difference method is introduced for comparison to the

finite element method. A general procedure in the sequence of model definition, solution, and interpretation of results is discussed and related to the generally accepted terms of preprocessing, solution, and postprocessing. A brief history of the finite element method is included, as are a few examples illustrating application of the method.

**Chapter 2** introduces the concept of a finite element stiffness matrix and associated displacement equation, in terms of interpolation functions, using the linear spring as a finite element. The linear spring is known to most undergraduate students so the mechanics should not be new. However, representation of the spring as a finite element *is* new but provides a simple, concise example of the finite element method. The premise of spring element formulation is extended to the bar element, and energy methods are introduced. The first theorem of Castigliano is applied, as is the principle of minimum potential energy. Castigliano's theorem is a simple method to introduce the undergraduate student to minimum principles without use of variational calculus.

**Chapter 3** uses the bar element of Chapter 2 to illustrate assembly of global equilibrium equations for a structure composed of many finite elements. Transformation from element coordinates to global coordinates is developed and illustrated with both two- and three-dimensional examples. The direct stiffness method is utilized and two methods for global matrix assembly are presented. Application of boundary conditions and solution of the resultant constraint equations is discussed. Use of the basic displacement solution to obtain element strain and stress is shown as a postprocessing operation.

**Chapter 4** introduces the beam/flexure element as a bridge to continuity requirements for higher-order elements. Slope continuity is introduced and this requires an adjustment to the assumed interpolation functions to insure continuity. Nodal load vectors are discussed in the context of discrete and distributed loads, using the method of work equivalence.

Chapters 2, 3, and 4 introduce the basic procedures of finite-element modeling in the context of simple structural elements that should be well-known to the student from the prerequisite mechanics of materials course. Thus the emphasis in the early part of the course in which the text is used can be on the finite element method without introduction of new physical concepts. The bar and beam elements can be used to give the student practical truss and frame problems for solution using available finite element software. If the instructor is so inclined, the bar and beam elements (in the two-dimensional context) also provide a relatively simple framework for student development of finite element software using basic programming languages.

**Chapter 5** is the springboard to more advanced concepts of finite element analysis. The method of weighted residuals is introduced as the fundamental technique used in the remainder of the text. The Galerkin method is utilized exclusively since I have found this method is both understandable for undergraduate students and is amenable to a wide range of engineering problems. The material in this chapter repeats the bar and beam developments and extends the finite element concept to one-dimensional heat transfer. Application to the bar

and beam elements illustrates that the method is in agreement with the basic mechanics approach of Chapters 2–4. Introduction of heat transfer exposes the student to additional applications of the finite element method that are, most likely, new to the student.

**Chapter 6** is a stand-alone description of the requirements of interpolation functions used in developing finite element models for *any* physical problem. Continuity and completeness requirements are delineated. Natural (serendipity) coordinates, triangular coordinates, and volume coordinates are defined and used to develop interpolation functions for several element types in two- and three-dimensions. The concept of isoparametric mapping is introduced in the context of the plane quadrilateral element. As a precursor to following chapters, numerical integration using Gaussian quadrature is covered and several examples included. The use of two-dimensional elements to model three-dimensional axisymmetric problems is included.

**Chapter 7** uses Galerkin's finite element method to develop the finite element equations for several commonly encountered situations in heat transfer. One-, two- and three-dimensional formulations are discussed for conduction and convection. Radiation is not included, as that phenomenon introduces a nonlinearity that undergraduate students are not prepared to deal with at the intended level of the text. Heat transfer with mass transport is included. The finite difference method in conjunction with the finite element method is utilized to present methods of solving time-dependent heat transfer problems.

**Chapter 8** introduces finite element applications to fluid mechanics. The general equations governing fluid flow are so complex and nonlinear that the topic is introduced via ideal flow. The stream function and velocity potential function are illustrated and the applicable restrictions noted. Example problems are included that note the analogy with heat transfer and use heat transfer finite element solutions to solve ideal flow problems. A brief discussion of viscous flow shows the nonlinearities that arise when nonideal flows are considered.

**Chapter 9** applies the finite element method to problems in solid mechanics with the proviso that the material response is linearly elastic and small deflection. Both plane stress and plane strain are defined and the finite element formulations developed for each case. General three-dimensional states of stress and axisymmetric stress are included. A model for torsion of noncircular sections is developed using the Prandtl stress function. The purpose of the torsion section is to make the student aware that all torsionally loaded objects are not circular and the analysis methods must be adjusted to suit geometry.

**Chapter 10** introduces the concept of dynamic motion of structures. It is not presumed that the student has taken a course in mechanical vibrations; as a result, this chapter includes a primer on basic vibration theory. Most of this material is drawn from my previously published text *Applied Mechanical Vibrations*. The concept of the mass or inertia matrix is developed by examples of simple spring-mass systems and then extended to continuous bodies. Both lumped and consistent mass matrices are defined and used in examples. Modal analysis is the basic method espoused for dynamic response; hence, a considerable amount of

text material is devoted to determination of natural modes, orthogonality, and modal superposition. Combination of finite difference and finite element methods for solving transient dynamic structural problems is included.

The appendices are included in order to provide the student with material that might be new or may be “rusty” in the student’s mind.

**Appendix A** is a review of matrix algebra and should be known to the student from a course in linear algebra.

**Appendix B** states the general three-dimensional constitutive relations for a homogeneous, isotropic, elastic material. I have found over the years that undergraduate engineering students do not have a firm grasp of these relations. In general, the student has been exposed to so many special cases that the three-dimensional equations are not truly understood.

**Appendix C** covers three methods for solving linear algebraic equations. Some students may use this material as an outline for programming solution methods. I include the appendix only so the reader is aware of the algorithms underlying the software he/she will use in solving finite element problems.

**Appendix D** describes the basic computational capabilities of the FEPC software. The FEPC (FEPfinite element program for the PCpersonal computer) was developed by the late Dr. Charles Knight of Virginia Polytechnic Institute and State University and is used in conjunction with this text with permission of his estate. Dr. Knight’s programs allow analysis of two-dimensional programs using bar, beam, and plane stress elements. The appendix describes in general terms the capabilities and limitations of the software. The FEPC program is available to the student at [www.mhhe.com/hutton](http://www.mhhe.com/hutton).

**Appendix E** includes problems for several chapters of the text that should be solved via commercial finite element software. Whether the instructor has available ANSYS, ALGOR, COSMOS, etc., these problems are oriented to systems having many degrees of freedom and not amenable to hand calculation. Additional problems of this sort will be added to the website on a continuing basis.

The textbook features a Web site ([www.mhhe.com/hutton](http://www.mhhe.com/hutton)) with finite element analysis links and the FEPC program. At this site, instructors will have access to PowerPoint images and an Instructors’ Solutions Manual. Instructors can access these tools by contacting their local McGraw-Hill sales representative for password information.

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